The remaining essay of social life, "Woman in Temple and Synagogue," is a useful supplement to Kayserling's more elaborate treatise, *Die Jüdischen Frauen in der Geschichte*, and contains original matter which is most interesting.

There are few books of which it can truly be said that they fill lacunae which were hitherto void, but I think I may safely say that Mr. Schechter's work deals in an interesting, instructive, and attractive manner with a number of subjects concerning which there are very few other sources of information in the whole range of English literature.

ALEX. MACALISTER.

THE ARITHMETIC OF ELIJAH MIZRAḤI.

Die Arithmetik des Elia Misrachi; ein Beitrag zur Geschichte der Mathematik, von GUSTAV WERTHEIM. Braunschweig, 1896.

THE custom of continental colleges to have a learned essay, written by one of the teaching staff, added to the annual report, encourages the teachers to continued study and research, each in his particular speciality, and has been the source of many valuable contributions to literature and science. From the same source comes the interesting book, *On the Arithmetic of Elijah Mizraḥi*, by Gustav Wertheim, which first appeared in the *Programm der Realschule der Israelitischen Gemeinde zu Frankfurt a. M.* 1893. The present volume is the second, improved and amplified, edition of this essay. Elijah Mizraḥi (b. 1455, d. 1526 at Constantinople) is well known to the student of Hebrew literature as the author of a supercommentary on Rashi's Commentary on the Pentateuch. When any difficulty is met with in Rashi, the Mizraḥi is consulted, and is generally found to discuss the problem in full length, though not always in a manner satisfactory to the puzzled inquirer. The reader of the supercommentary will hardly expect that the author was an excellent mathematician, because he never avails himself of any given opportunity to display his knowledge of astronomy, geometry, or arithmetic. As Chief Rabbi of the Jewish congregation at Constantinople, he had frequently to reply to questions addressed to him on religious matters, and some of the replies are contained in two collections of Responsa, viz. *Mayim Amukim* and *Shaaloth utheshubhoth*. But the work to which our attention is for the present directed is his *Sefer ha-Mispar* (The Book of Arithmetic), of which Rabbi Joseph Solomon del Medigo, in a letter addressed to his son, says: "It is indeed a very valuable book to those who are able to

read it and to understand it" (Geiger's *Melo Chofnayim*, p. 12). Mizraḥi considers mathematics as the intermediary between theology and natural science, being, as it were, the bridge which leads thought from the material to the immaterial, and enables men to obtain a thorough knowledge of things. Of the four parts of mathematics — arithmetic, geometry, astronomy, and music—the first two are of a more general character, and must in study precede the other two; and since arithmetic has frequently to be applied in geometry, it must be studied first of all the different branches of mathematics. There was no lack of books on arithmetic; but these were unsatisfactory in the opinion of Mizraḥi, because they only taught how to solve arithmetical problems without showing the reason for the method adopted. The students, in using such books, did not learn the method adopted by other scholars, and much less were they trained in the art of inventing new methods of their own, when they met with problems different from those which they were taught. And as, in addition, his numerous disciples had begged him to write for them a book on arithmetic, and discuss in it the methods of previous authors with their proofs and arguments, he resolved to comply with their request, and his *Sefer ha-'Mispar* is the outcome of this resolve. He did not, however, pretend to give all possible methods, but he promises to teach the foundations of the various artifices which are employed by teachers of arithmetic in the solution of problems. Of the numerous interesting problems discussed by Mizraḥi, the following will suffice to show his genius and method.

1. Find the sum of the natural numbers from 1 to n.

Solution.—The proportion of 1 to the next number is $=\frac{1}{2}$; of $1+2$ to the next number is $=1$, i.e. $\frac{1}{2}$ more than the preceding; of $1+2+3$ to the next number is $=\frac{3}{2}$, again $\frac{1}{2}$ more than the preceding proportion, and so on,

hence $$\frac{1+2+3+\ldots+(n-1)+n}{(n+1)}=n \cdot \tfrac{1}{2}$$

and $$1+2+3+\ldots+n=(n+1)n \cdot \tfrac{1}{2}$$

2. Find the sum of the squares of the natural numbers from 1 to n.

Solution.—Comparing the sum of the squares of the natural numbers with the sum of these numbers themselves, we obtain the following equations:—

$$\frac{1^2}{1}=1; \qquad \frac{1^2+2^2}{1+2}=1+\tfrac{2}{3}; \qquad \frac{1^2+2^2+3^2}{1+2+3}=1+2 \cdot \tfrac{2}{3};$$

$$\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=1+3 \cdot \tfrac{2}{3};$$

and so on, each successive proportion increasing by $\frac{2}{3}$; so that

$$\frac{1^2+2^2+3^2+\ldots+n^2}{1+2+3+\ldots+n}\text{is}=1+(n-1).\tfrac{2}{3}=\frac{2n+1}{3}$$

Hence
$$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{2\cdot 3}$$

3. Find the sum of the cubes of the natural numbers.
Solution.—

$$\frac{1^3}{1}-0=1$$

$$\frac{1^3+2^3}{1+2}-\frac{1^3}{1}=2$$

$$\frac{1^3+2^3+3^3}{1+2+3}-\frac{1^3+2^3}{1+2}=3$$

$$\frac{1^3+2^3+3^3+4^3}{1+2+3+4}-\frac{1^3+2^3+3^3}{1+2+3}=4$$

$$\frac{1^3+2^3+3^3+\ldots+n^3}{1+2+3+\ldots+n}-\frac{1^3+2^3+3^3+\ldots+(n-1^3)}{1+2+3+\ldots+(n-1)}=n$$

By addition we obtain

$$\frac{1^3+2^3+3^3+\ldots+n^3}{1+2+3+\ldots+n}=1+2+3+4+\ldots+n^2$$

and
$$1^3+2^3+3^3+\ldots+n^3=(1+2+3+4+\ldots+n)$$

Among the works used by Mizraḥi the *Sefer ha-Mispar* of Ibn Ezra occupies the first place. Of the hundred problems treated in Mizraḥi's book, twenty-one are taken *verbatim* from Ibn Ezra. Also in the theoretical part the same source may frequently be traced. It is remarkable that Mizraḥi, like Ibn Ezra (*J. Q. R.* IX. p. 659), ignores altogether problems concerning interest and discount.—Herr Wertheim gives only an analysis of Mizraḥi's work, but this analysis, being concise and clear, will prove far more useful than a literal translation, which in many cases is less intelligible to the reader than the original. In the interest of Hebrew literature, however, I should have liked to see this analysis as an introduction to an edition of the Hebrew text. Sebastian Münster, of the sixteenth century, found this work of such importance and usefulness that he edited for his pupils a part of it, together with a portion of Abraham b. Ḥiyya's *Tsurath ha-Arets*; we may fairly assume that books of this kind will also in the present century, with its numerous new centres for the study of Hebrew literature, find friends, readers, and admirers.

M. FRIEDLÄNDER.